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The employability of an aging population in a world of continuous technical change is top of the political agenda. Due to endogenous human capital depreciation, the effective retirement age is often below statutory retirement age resulting in unemployment among older workers. We analyze this phenomenon in a putty-putty human capital vintage model and focus on education and the speed of human capital depreciation. Introducing a two-stage education system with initial schooling and lifelong learning, not even lifelong learning turns out to be capable of aligning economic and statutory retirement. However, lifelong learning can increase the number of people reaching statutory retirement age and hence reduce the problem of old age unemployment in an aging society.

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JEL Classification: J26, O33, J64.

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The employability of an aging population in a world of continuous technical change is top of the political agenda. Due to endogenous human capital depreciation, the effective retirement age is often below statutory retirement age resulting in unemployment among older workers. We analyze this phenomenon in a putty-putty human capital vintage model and focus on education and the speed of human capital depreciation. Introducing a two-stage education system with initial schooling and lifelong learning, not even lifelong learning turns out to be capable of aligning economic and statutory retirement. However, lifelong learning can increase the number of people reaching statutory retirement age and hence reduce the problem of old age unemployment in an aging society.

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1 Introduction

In many OECD countries the effective retirement age is well below the statutory retirement age. The major reason for the low labor-force participation of the 55 to 64 age group are high net withdrawal rates, i.e. a major reduction in employment figures when moving to the next higher age group. At the same time, many governments consider extending working life as the dominant strategy for keeping the pension system sustainable,[1] while keeping the standard of living (almost) constant because individuals continue to earn an income. This argument, however, depends strongly on the supply of and the demand for older workers. It implicitly presumes that workers remain employable until the new, higher retirement age. However, if the effective, or rather economic, retirement age does not increase when the statutory retirement age is raised, old-age poverty of large groups in society could be the result.

For instance, Butrica et al. (2002) show that raising the statutory retirement age will contribute strongly to the projected rise in old-age poverty in the US.[2] Likewise, extended periods of unemployment are seen as a major explanation for old-age poverty (see e.g., Smeeding, 1999, 2001; Gottschalk, 2006; or Favreault and Steuerle, 2008). In the context of pension reform, increasing the employability of older people is therefore highly important. As Holzmann (2005) argues, the employability of older workers calls for changes in the wage profile and for measures to maintain and raise their productivity, especially the implementation of lifelong learning.

There is no doubt that a pension system's fiscal impact plays an important role in keeping labor-force participation of older workers low, for example by early-

[1] See e.g., Barr (2004, p. 209) or Cremer and Pestieau (2003) who even expect a "double dividend" from raising the statutory retirement age. Based on a OLG-CGE model, Marchiori (2008) explicitly considers raising the retirement age for unskilled workers in Europe to be the most effective measure for relieving fiscal pressure.

[2] Considering above-NAIRU economies, Michello and Ford (2006) argue that increasing retirement age will directly add to unemployment.

retirement schemes (see e.g., Gruber and Wise, 1999; or Fenge and Pestieau, 2006). However, according to Duval (2003) only about one third of the decline in the older males' labor-force participation rate can be attributed to implicit tax rates and retirement age.[3] The rest stems from demand-side factors and preferences for leisure. It is often claimed that the majority of older workers cannot choose to work until the age of 67 or beyond as there is insufficient demand for this type of labor. Standard arguments are costly seniority rules (Lazear 1979, 1981) and youth unemployment, which is hoped to be overcome by pushing older workers out of work (see the discussion in Boldrin et al., 1999). Fenge and Pestieau (2006, p. 107) point out four potential explanations for this effect:

- Low-skilled older workers may exhibit declining relative productivity in times of rapid technological change.
- Insufficient training may contribute to declining productivity among older workers.
- Rigid age-earnings profiles caused by specific institutional arrangements (e.g. employment-protection legislation) reduce employment opportunities for the older unemployed.
- Temporary negative demand shocks may lead to an irreversible labor force withdrawal.

In this paper we focus on the first two propositions. In today's fast-changing knowledge economy people need to upgrade their skills – in both the personal and professional domain – throughout their adult lives to cope with modern life (OECD, 2007). Only this guarantees that they will remain employable for the duration of their working life and beyond. In fact, several studies observe a growing importance of certain general and cognitive skills in the labor market.[4] Not surprisingly,

[3] Johnson's (2000) estimate of this measure is even lower at 11 per cent.

[4] See e.g., Gould (2002), Weinberg (2000), Murname et al. (1995) or Taber (2001). According to Bishop (1991), however, the results are not as clear cut.

lifelong learning for all has become a widely shared policy objective among industrialized countries; a striking example being the European Union (starting with the Lisbon Strategy) which concluded that a "need for a continuous renewal of citizens' knowledge, skills and competencies is crucial for the EU's competitiveness and social cohesion" (EU Commission, 2006). One central focus of our model will be the question whether investing into general education or lifelong learning, respectively, will indeed help to synchronize the economic and the statutory retirement age.

Over the past thirty years, in almost all industrialized countries the demand for labor has shifted in favor of skilled workers. There is substantial evidence that skill-biased technological progress and the globalization of production are among the main driving forces of this shift in demand.[5] As already mentioned in the OECD Jobs Strategy (OECD, 1994), many industrialized countries have responded by taking measures to improve the education of the workforce.[6] Since older workers are particularly affected by technologically induced skill depreciation,[7] adult training and lifelong learning are an essential part of these programs. As the OECD (2004) finds, "for older and low-educated workers, training allows attaining and maintaining the competencies required to bring productivity in line with market wages, thereby sustaining employment prospects of these groups" (OECD, 2004, p. 185). From a theoretical perspective, standard arguments are that these competencies should be sufficiently general to facilitate adjustment to technological change (Welch, 1970) and/or to future shocks as only this reduces uncertainty such that lower human-capital investment is avoided (Lehvari and Weiss, 1974).

Despite these efforts, the empirical evidence on returns to training and adult learning is still inconclusive (OECD, 2004). In a study based on cross-sectional aggregate data and longitudinal surveys, the OECD (2004) finds strong evidence of

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[5] See Chennells and van Reenen (2002) for a survey of literature on skill biased technological progress, OECD (1997) for employment effects of internationalisation, and Nickell and Bell (1996) for the impact on unemployment.

[6] See e.g. OECD (1994) for an analysis and OECD (2003) for related policy recommendations.

[7] See OECD (2004), Ch. 4 and the references therein.

positive effects of education and training on employment outcomes at the individual level. These effects are due to higher participation rates and a lower probability of unemployment. The positive correlation between education and participation rates is also evident at the aggregate level. However, there is still no empirical support for a positive effect of training on aggregate unemployment rates (see OECD, 2004, p. 189).

Taking these stylized facts from the labor market as a point of departure we focus on the production process and analyze the depreciation of human capital. We obtain conditions for the economic retirement age of an aging workforce. In our approach workers are heterogeneous. While all workers enjoy access to the same education, - namely *initial schooling* and thereafter a *lifelong learning program* -, the distribution of learning capabilities affects workers in different ways. For a given level of spending into a lifelong learning program a fraction of the workforce becomes adaptable to new generations of technologies. For the remainder the present lifelong learning program is not successful; hence they are left with vintage specific skills acquired during *initial schooling*. These are subject to economic depreciation as the generations of technologies age. We choose a vintage model because technological change affects existing and new production processes asymmetrically.[8] To avoid results that are simply due to a limitational production structure, the vintage model is of the putty-putty type. In a putty-putty vintage model, production factors can be substituted at the time when the technology is installed and also when it is in use.

The analysis will show that technological progress leads to a specific employment structure. Due to technological progress, the most recent vintages of adaptable workers are most productive. Therefore, the demand and input of adaptable workers decline as vintages age. Accordingly, due to cross effects of production factors the productivity of vintage specific skills in older vintages decreases as well. The depreciation of vintage specific human capital endogenously determines the economic

[8]The application of vintage models to labor markets, demographic developments and social security systems was recently suggested by Mateos-Planas (2001), Boucekkine, de la Croix and Licandro (2002), Echeveria and Iza (2006), and Hornstein, Krusell and Violante (2007).

retirement age of the respective workers. If economic falls short of statutory retirement age, old age unemployment results. Even fully flexible wages will not be able to generate market equilibrium in the labor market segment of older workers. While labor with adaptable technological competencies is able to switch to any technology efficiently, labor using vintage specific skills is discarded no matter what wage they would agree to work for.

Although technological progress is most important for the human-capital depreciation process, it is also the most difficult to influence. Improving education is often suggested to increase the productivity of aging labor and thus to keep labor employed until the statutory retirement age is reached. However, while this appears plausible at a first glance, it is not the outcome of our theoretical analysis. We distinguish between two stages of education, initial schooling and lifelong learning and all workers have to undergo both stages of education. Depending on their educational success they will obtain adaptable or vintage specific skills. With respect to the effects of education policy we find that:

- An improvement in *initial schooling*, that is, education before working life starts, will affect income levels and the distribution of income. However, spending on initial education has no effect on the depreciation rate of human capital and the economic retirement age of inflexible workers.

- Whether *lifelong learning programs* can help to adjust the speed of the depreciation process partly depends on the substitutability of different types of workers. However, even if *lifelong learning programs* fail to solve the problem of synchronizing economic and statutory retirement age they still help to increase the individual probability of finding a job when people grow older and hence reduces the unemployment rate of an aging generation.

The paper proceeds as follows: In Section 2 we will introduce a model of endogenous depreciation of human capital. In Section 3 the model is solved for the steady state. To show the effects of the putty-putty technology with different elasticities of substitutions ex ante and ex post, the development of labor demand and the output of a given technology vintage is discussed. In Section 4 the results and implications with respect to the retirement age and education systems are presented and in Section 5 we conclude.

2 A Human Capital Vintage Model

Vintage models were introduced in the 1950s and 60s to analyze the effects of technological progress. The complexity and implications of the technical diffusion process have recently encouraged a number of authors to apply a vintage approach with embodied technologies in growth models.[9] In general, vintage models are used where aggregate (closed) production functions with disembodied technologies do not seem to be an appropriate instrument to model the interaction of newly implemented capital, existing capital and new technologies. The most important difference to a closed form aggregate production function is that the substitution process of factors is relatively more difficult for already installed production processes compared to currently implemented processes.

In the real world technological advances are often implemented via new investments. Running these new technologies often requires an adaptation of production processes and skills. As technical progress is not just an invention of a technology but also a diffusion through the production process, education and the structure of education plays an important role for a society's ability to implement technical progress. These ideas, technology adoption and the role of education first developed by Stephens (1971), van Imhoff (1988), or Chari and Hopenhayn (1991), were re-

[9]E.g. Boucekkine and Pommeret (2004) discuss the implementation of energy saving technologies when energy prices increase. Boucekkine, Licandro, Puch and del Rio (2005) introduce the vintage approach into the AK-model of endogenous growth, illustrating that introducing vintage capital changes the dynamics of a growth process substantially. Optimal investment strategies under different external conditions and expectations are considered by Gilchrist and Williams (2005), Feichtinger et al. (2006) or Goetz et. al. (2008).

cently taken up again. Mateos-Planas (2001) considers the effectiveness of schooling and technology specific learning-by-doing. Boucekkine, de la Croix and Licandro (2002) define generation specific human capital and analyze demographic phenomena (like change in life expectancy) in an endogenous growth OLG setting. In this model each generation is tied to its own vintage technology and accumulates pure vintage specific human capital. The human capital vintage structure is regarded as an important ingredient to cover the aging effects of a non homogeneous human capital stock for endogenous education and retirement decision as well as for per capita growth. With a model similar to Boucekkine et al. (2002) Echevaria and Iza (2007) include social security effects on education and retirement decision and analyze how life expectancy affects per capita growth. The problem of skill transferability between subsequent technology generations and the effects on the labor market are addressed by Violante (2002). One mechanism of his analysis is based on vintage specific human capital. When separating in a labor market matching model workers can only partially transfer their skills to machines. This limited flexibility due to vintage specific human capital increases wage losses upon separation. A matching model with vintage specific human capital was built by Hornstein et al. (2007). The paper addresses changes in labor demand due to changes in technology. In this model the focus lies on frictions and the labor market matching process. The vintage structure of production contributes to the matching problem. Rather than emphasizing the firms' costs of "posting vacancies", the paper stresses the upfront, irreversible costs of investing in equipment as the typical vintage model. As a result capital-embodied technological change reduces labour demand and raises equilibrium unemployment and unemployment duration.

Based on the above discussion we assume that technical progress can only be implemented if an adjustment in skills enables the use of a new generation (vintage) of technology. Moreover, Mateos-Planas (2001) identified two hypotheses about the effects of education in the literature. One postulates that education provides a productivity advantage which is independent of experience. Education not only has a productivity effect and generates human capital. According to the other hypothesis, education improves the ability of agents to adapt new technologies, hence education generates technological flexiblitiy. Only this education effect makes it possible to combine new technologies with sufficiently educated labor to handle it. Hence, there is an important interaction between the implementation of new technologies and a reallocation in the labor market. The schooling system in our model addresses both aspects of education.

Schooling system: To determine domestic labor input, we need to characterize the workforce. The schooling process is modeled as a sequence of two education cycles. *Initial schooling* is followed by a *lifelong learning program.* All workers N_V^s of each vintage V have to participate in both education cycles.

Initial schooling equips workers with skills that enable them to use the most modern technology available at the time when a new generation of workers enters the market. Apart from providing a basic educational grounding, these skills lead to a human capital endowment of the workers that is related only to the most recent technology vintage introduced.

Starting with *initial schooling* the human capital endowment of the workforce of each vintage is determined by the rate of income τ_1 spent on *initial schooling* . Assuming a minimum required level $\tau_1^{\min}$, *initial schooling* is supposed to be equally successful for all and hence equally spread over the whole workforce.[10] The vintage specific human capital endowment of each worker

$$h_V(t,t) = \frac{\tau_1 Y(t)}{N_V^s}, \tag{1}$$

where $Y(t)$ denotes aggregate income. Therefore, after initial schooling each worker is endowed with a human capital endowment $h_V(t,t)$ tied to the technology vintage

[10]This is the "neutrality hypothsis" in Mateos-Planas (2001). See also Lucas (1988), Grossman and Helpman (1991, Ch. 5) and others.

of the period of his initial education.[11] This basic human capital endowment is the asset the worker can offer on the labor market at any time.

In this model *lifelong learning* is a resource-absorbing *schooling program.* With a *lifelong learning program* workers will be instructed to adjust to each following generation (most recent vintage) of technologies. Successful attendance in lifelong learning programs allows workers to adapt to the newest technology.[12] Hence successful attendance in a *lifelong learning program* enables to switch between all technology vintages installed. An established *lifelong learning program* helps aging workers to adapt to changing technologies.[13]

If for a worker a *lifelong learning program* is not successful he still has his ability of using and handling the technology vintage he originally learned to use when he once entered the labor market as a young person.

While we assume that *initial schooling* is equally successful for all workers, the effects of the *lifelong learning program* are more complex. In fact we assume that the domestic labor force is heterogeneous in terms of learning abilities. Therefore, even if all workers undergo both types of education, the success of the *lifelong learning program* depends on the distribution of the population's learning capabilities.

Let q denote the average probability that the lifelong learning program is successful. Further, we assume that q is determined by the resources spent on this program ($R_{II} = \tau_2 Y$) and the distribution of the population's learning capabilities:

[11] Concerning this assumption Boucekkine, de la Croix and Licandro (2002) argue on page 343 that "... different generations have different learning experiences and ... the aggregate stock of human capital is built from the human capital of different generations. The most important characteristic of a growth theory designed to shed light on these issues is clearly to capture the *vintage nature of human capital.*"

[12] This is the "learning hypothsis" in Mateos-Planas (2001) which refers to an education concept as suggested by Nelson and Phelps (1966), Welch (1970) and others.

[13] With this assumption we depart from Boucekkine, de la Croix and Licandro (2002) who define pure vintage specific human capital and do not allow a switch from one to another technology vintage.

$$q = q(R_{II}) = q(\tau_2 Y).$$

Therefore, the more resources spent on *lifelong learning programs* each year, the more likely the success and the ability of workers to adapt to new technologies ($\frac{\partial q}{\partial R_{II}} > 0$). Further, heterogeneous abilities of the workforce cause the marginal effectiveness of additional spending to decrease. Thus, even if all workers attend the *lifelong learning program* the increase in the probability of success of such a program decreases with an increasing level of R_{II} ($\frac{\partial^2 q}{\partial R_{II}^2} < 0$). Hence we can determine the expected number of technologically adaptable workers of a vintage

$$EN^s_{V,II} = qN^s_V.$$

For the remaining fraction of workers the investments of the lifelong learning program were not sufficient to generate an ability to switch to the most recent vintage of technologies. The expected number of these workers is

$$EN^s_{V,I} = N^s_V - EN^s_{V,II} = (1-q)\, N^s_V.$$

We will skip the E henceforth, as we are interested only in the expected value. Even if these workers can still offer their existing human capital during initial schooling, their human capital is connected to the vintage specific technology and hence remains a vintage specific human capital $h_V(t,t)$ throughout their entire economic life.

Labor market: The effects of the lifelong learning program together with the distribution of abilities among the population has created two labor market segments.

First, the two education cycles lead to a workforce offering vintage specific skills. Education efforts for lifelong learning were not sufficiently effective to enable these $N^s_{V,I}$ workers to adapt other technologies than the ones they learned to use during *initial schooling.* Therefore, these workers continue to use the same technology throughout their working life. The human capital level of a whole vintage is the human capital endowment multiplied by the number of workers using *vintage specific skills*:

$$H_V(t,t) = h_V(t,t)N_{V,I}.$$

While in traditional vintage models the technology can be represented by real capital, here vintage specific human capital characterizes a certain production process and technology. Assuming again perfectly competitive markets, a vintage of this type of labor is either in full demand or fully unemployed; it is either $N^d_{V,I} = N^s_{V,I} = N_{V,I}$ or $N^d_{V,I} = 0$. With $m(t)$ generations of technologies in use, we obtain a demand for $m(t)$ vintages of labor with vintage specific skills. Denoting T as the statutory retirement age the aggregate demand and aggregate supply of workers using vintage specific skills is

$$N^d_I = \int_{t-m}^{t} N_{V,I}dv \quad \text{and} \quad N^s_I = \int_{t-T}^{t} N_{V,I}dv.$$

Obviously, this kind of labor is fully employed only if $T \leqq m(t)$. In terms of retirement, this implies that the worker retires at the statutory retirement age T.

Second, another segment of the labor market consists of adaptable labor. For these workers $N^s_{V,II}$ the present level of spending on the lifelong learning program was successful. If the lifelong learning program is successful, a worker is able to adapt new technologies and hence can switch to any new generation of technologies efficiently. Thus technological progress λ_V augments this type of labor. At time t labor with flexible competencies employed in the technology vintage v in efficiency units is

$$\Lambda_V(v,t) = \lambda_V(v)N^d_{V,II}(v,t).$$

We assume that technological progress grows at the given rate $\hat{\lambda} := \frac{\dot{\lambda}_V}{\lambda_V}$.[14] Since technological progress is embodied, λ_V will affect only the current production vin-

[14]See de Mello (1995) or Boucekkine, Licandro, Puch and del Rio (2005) for endogenous growth conditions in vintage models.

tage and not existing production processes. This is a major difference of vintage technologies compared to simple aggregate production functions.
Adaptable labor is demanded by the newest $m(t)$ technologies. With T as the total economic lifetime (given by the statutory retirement age), labor supply consists of T vintages. Thus, aggregate demand and aggregate supply of flexible labor is

$$N_{II}^d = \int_{t-m}^{t} N_{V,II}^d(v,t)dv \quad \text{and} \quad N_{II}^s = \int_{t-T}^{t} N_{V,II}^s(v)dv.$$

As we assume perfectly competitive markets, workers descending from T vintages will be in demand due to the newest technologies m. Thus, these workers will never be unemployed. Their retirement usually begins at the statutory retirement age, which is here equivalent to T.

Production: In a vintage approach the implementation of technical changes is more complex than defined by a closed (aggregate) production function. When introducing a vintage model we have to consider the substitution possibilities ex post and ex ante. For a given vintage technology, the inputs of production can easily be substituted ex ante while there are limited substitution possibilities ex post. To avoid results that are due to fixed input relations we use a putty-putty vintage approach. Specifically, we assume that the elasticity of substitution ex ante is higher than the elasticity of substitution ex post. Thus, the model is a combination of the putty-putty approach first modeled by Solow (1960) and the putty-clay approach first modeled by Johansen (1959) and Salter (1960).[15] The putty-putty model below allows for substitution and also describes an obvious adaptation and adjustment problem generated by the technical requirements for adjusting technologies introduced in the past.

The production ex ante and ex post is modeled by a nested CES function. Ag-

[15]Putty-putty models with limited substitution possibilities ex post can be found in Scheper (1968), Zon (1994), and Meijers and Zon (1994), or Gilchrist and Williams (2005).

gregate output, Y, is the sum of vintage outputs, Y_V, producing:[16]

$$Y(t) = \int_{t-m}^{t} Y_V(\upsilon, t) d\upsilon, \tag{2}$$

where m is the age of the oldest technology in use. At time t vintage output Y_V of the vintage installed at υ is produced with imports Im and domestic labor services S. At the time of installing a new technology, ex ante, the elasticity of substitution is $\sigma_1 := \frac{1}{1+\rho_1}$ such that we arrive at

$$Y_V(t,t) = \left(\gamma Im(t,t)^{-\rho_1} + (1-\gamma)S(t,t)^{-\rho_1}\right)^{-\frac{1}{\rho_1}}. \tag{3}$$

The already installed production process is more difficult to adjust. Ex post the inputs cannot be substituted to the same extent as ex ante. Thus, the elasticity of substitution ex post ($\upsilon < t$) is $\sigma_2 = \frac{1}{1+\rho_2}$ with $\sigma_1 > \sigma_2$ and

$$Y_V(\upsilon,t) = \left(\gamma_\upsilon Im(\upsilon,t)^{-\rho_2} + \delta_\upsilon S(\upsilon,t)^{-\rho_2}\right)^{-\frac{1}{\rho_2}}. \tag{4}$$

The following two conditions ensure that the relation of factor productivity does not change for the technology after it has been installed:

$$\eta_{Y(t,t),S(t,t)} = \lim_{\upsilon \to t} \eta_{Y_V(t,\upsilon),S(t,\upsilon)}, \tag{5}$$

$$Y_V(t,t) = \lim_{\upsilon \to t} Y_V(\upsilon,t). \tag{6}$$

Formally, a continuous differentiable relation between the ex post and the ex ante production function is guaranteed. γ_υ and δ_υ are determined depending on the parameters γ and $1-\gamma$.

Labor services: The supply of labor services is modeled with a labor service production function. Here again a putty-putty technology with different elasticities of substitution ex ante and ex post is considered. The ex ante production of labor services is assumed to be of the Cobb-Douglas type:

$$S(t,t) = H_V(t,t)^{1-\alpha}\Lambda_V(t,t)^{\alpha}. \tag{7}$$

[16] We denote all variables belonging to a certain vintage with a subindex V.

The production inputs for the labor service are vintage specific human capital H_V and adaptable labor in efficiency units Λ_V. Installing a new technology, vintage specific human capital and flexible labor can easily be substituted. Thus, the elasticity of substitution is $\sigma_3 = 1$. It is also possible to substitute the inputs ex post. However, the elasticity of substitution ex post is assumed to be smaller than ex ante: $\sigma_4 := \frac{1}{1+\rho_4} < \sigma_3 = 1$. For the labor services we obtain

$$S(v,t) = \left(\alpha_v \Lambda_V(v,t)^{-\rho_4} + \beta_v H_V(v,t)^{-\rho_4}\right)^{-\frac{1}{\rho_4}} \tag{8}$$

for $v < t$. The following two conditions guarantee that the relation of productivity does not change when moving from ex ante to ex post service production:

$$\eta_{S(t,t),N_{VII}(t,t)} = \lim_{v \to t} \eta_{S(t,v),N_{V,II}(t,v)}, \tag{9}$$

$$S(t,t) = \lim_{v \to t} S(v,t). \tag{10}$$

Thus, α_v and β_v are determined endogenously.

Efficient factor allocation: With a putty-putty technology the efficient allocation of inputs can be divided into two parts. (i) The optimal human capital and the optimal import intensity are chosen by maximizing the profits of employing vintage specific human capital when installing a new technology. (ii) Since the production factors can be substituted ex post, the relation of the inputs is chosen optimally after installing a technology. While imports and adaptable workers are paid according to their marginal product, vintage specific human capital and the respective workers receive the residual income determined in a zero profit market equilibrium.

(i) Let p be the price of imports. Adopting the small country assumption, p is regarded as exogenous. The profits of employing vintage specific human capital are

$$\Pi = \int_t^{t+m} \left(Y_V(t,v) - pIm(t,v) - w(t)N_{V,II}^d(t,v)\right) e^{-r(v-t)} dv - H_V(t,t),$$

where r is the quasi-rent of vintage specific human capital and w the wage rate of adaptable labor. Writing this equation in intensity form we get ($\pi := \frac{\Pi}{N_{V,II}^d(t,t)}$, $y_V(t,v) :=$

$\frac{Y_V(t,v)}{N^d_{V,II}(t,v)}$, $\kappa(t,v) := \frac{H_V(t,v)}{N^d_{V,II}(t,v)}$, $im(t,v) := \frac{Im(t,v)}{N^d_{V,II}(t,v)}$):

$$\pi = \int_t^{t+m} \frac{N^d_{V,II}(t,v)}{N^d_{V,II}(t,t)} \left(y_V(t,v) - pim(t,v) - w(t)\right) e^{-r(v-t)} dv - \kappa(t,t).$$

The optimal vintage specific human capital intensity at the time of installation is

$$\frac{\partial \pi}{\partial \kappa(t,t)} = 0 \quad \Longleftrightarrow \quad \int_t^{t+m} \frac{N^d_{V,II}(t,v)}{N^d_{V,II}(t,t)} \left(\frac{\partial y_V(t,v)}{\partial \kappa(t,t)}\right) e^{-r(v-t)} dv = 1 \tag{11}$$

and the optimal import intensity is

$$\frac{\partial \pi}{\partial im(t,t)} = 0 \quad \Longleftrightarrow \quad \int_t^{t+m} \frac{N^d_{V,II}(t,v)}{N^d_{V,II}(t,t)} \left(\frac{\partial y_V(t,v)}{\partial im(t,t)} - p\frac{\partial im(t,v)}{\partial im(t,t)}\right) e^{-r(v-t)} dv = 0. \tag{12}$$

(ii) Having installed a technology the inputs are then inserted efficiently by maximizing the profits of a certain technology (v= installing time, t= present time)

$$\Pi_V(v,t) = Y_V(v,t) - pIm(v,t) - w(t)N^d_{V,II}(v,t) - r_V(v,t)H_V(v,t).$$

r_V is the vintage rental rate of vintage specific human capital determining the wage of the respective labor of vintage specific labor. Since human capital is tied to a certain vintage and no internal depreciation of human capital is considered, we have $H_V(v,t) = H_V(v,v)$. In contrast adaptable labor and imports inserted in a certain technology can change after installing a technology. Thus, the following two profit maximization conditions follow. Imports are allocated efficiently ex post such that for $v < t$ we obtain[17]

$$\frac{\partial \Pi_V(v,t)}{\partial Im(v,t)} = 0 \quad \Longleftrightarrow \quad \frac{\partial Y_V(v,t)}{\partial Im(v,t)} = p. \tag{13}$$

Workers with adaptable technological skills are also allocated efficiently ex post:

$$\frac{\partial \Pi_V(v,t)}{\partial N^d_{V,II}(v,t)} = 0 \quad \Longleftrightarrow \quad \frac{\partial Y_V(v,t)}{\partial N^d_{V,II}(v,t)} = w(t). \tag{14}$$

[17] For $v = t$ this condition is equivalent to equation (12) – see Corollary 5.

The rental rate of vintage specific human capital is determined by the zero-profit condition:

$$\Pi_V(v,t) = 0 \quad \Longleftrightarrow \quad r_V(v,t)H_V(v,t) = Y_V(v,t) - pIm(v,t) - w(t)N^d_{V,II}(v,t). \tag{15}$$

Using Euler's theorem (15) can be written as

$$\frac{\partial \Pi_V(v,t)}{\partial H_V(v,t)} = 0 \quad \Longleftrightarrow \quad \frac{\partial Y_V(v,t)}{\partial H_V(v,t)} = r_V(v,t).$$

The scrapping rule is straightforward. A vintage is discarded after m vintages if the profits of the vintage vanish (see Benhabib and Rustichini, 1993, and van Hilten, 1991, for scrapping rules) i.e. the income of vintage specific human capital is zero, if $r_V(t-m,t) = 0$ or

$$Y_V(t-m,t) - pIm(t-m,t) - w(t)N^d_{V,II}(t-m,t) = 0. \tag{16}$$

This scrapping rule determines the economic scrapping time m of a technology vintage.

3 Solution of the Model

We solve the model for the steady state.[18] To show the effects of the putty-putty technology, we focus on the development of the variables of one vintage. Equations (2)-(16) determine the growth path of the endogenous variables. Due to its structure, the model can be written in intensity form. The economy is in steady state if the demand for adaptable labor of a new technology $N^d_{V,II}(t,t)$ is constant – we do not consider population growth – and vintage specific human capital and imports grow at the rate of technological progress $\hat{\lambda}$. It follows that the vintage output $y_V(t,t)$, the aggregate output y,[19] domestic labor services $s(t,t)$, and the wage rate of

[18]For the convenience of the reader all relevant theorems and lemmas are presented in the appendix.

[19]See Lemma 9.

adaptable labor[20] grow at the rate λ, while the scrapping time m,[21] the coefficients[22] $\delta_t, \gamma_t, \alpha_t, \beta_t$ as well as the quasi-rent[23] r are constant in steady state.

Formally the model is solved in three steps:

First, the scrapping time m^* is determined. A vintage is discarded if the rental rate of human capital of a vintage $r_V(t, t+m^*) H_V(t,t) = Y_V(t, t+m^*) - pIm(t, t+m^*) - w(t+m^*) N^d_{V,II}(t, t+m^*)$ is zero. The condition is determined by the structure of the growth rates in steady state. m^* is not dependent on the level of the output or the intensities of the model. Thus, the scrapping time is[24]

$$m^* = \frac{\ln\left[\left(1 - \left(\frac{\gamma}{p}\right)^{\frac{1}{1+\rho_1}}\right)^{-1} \alpha\right]}{-\hat{\lambda}} \frac{1+\rho_4}{\rho_4}. \tag{17}$$

Second, to obtain the remaining variables human capital intensity and the quasi-rent have to be determined. Depending on κ the growth path of the variables follows immediately.[25] Human capital intensity and the quasi-rent of vintage specific human capital are determined through the supply of and demand for vintage specific human capital given in equations (1) and (11). In theorem 10 it is shown that unique values of κ and r follow if the rate of *initial schooling* is not too high ($\tau_1 < \tau_1^{max}$).[26]

Third, the development of the variables of a certain vintage are determined: the demand for adaptable labor, the vintage output of a certain technology, and the rental rate of vintage specific human capital.

The demand for adaptable labor of the technology installed in v at time t is[27]

$$N^d_{V,II}(v,t) = \left(\frac{\left(\frac{w(t)}{c_2 \alpha_v \lambda_V(v)^{-\rho_4}}\right)^{-\frac{\rho_4}{1+\rho_4}} - \alpha_v \lambda_V(v)^{-\rho_4}}{\beta_v}\right)^{\frac{1}{\rho_4}} H_V(t,t).$$

[20]See Corollary 3.

[21]See Theorem 6.

[22]See Lemma 1.

[23]See Theorem 10.

[24]See Theorem 6.

[25]See Lemma 2, Corollary 3 for the output, the imports and the wage rate of flexible labor. The rest follows from the definition of the variables and Lemma 1.

[26]If this condition does not hold, the quasi-rent r will be negative.

[27]Using Lemma 2 this condition follows from equation (14).

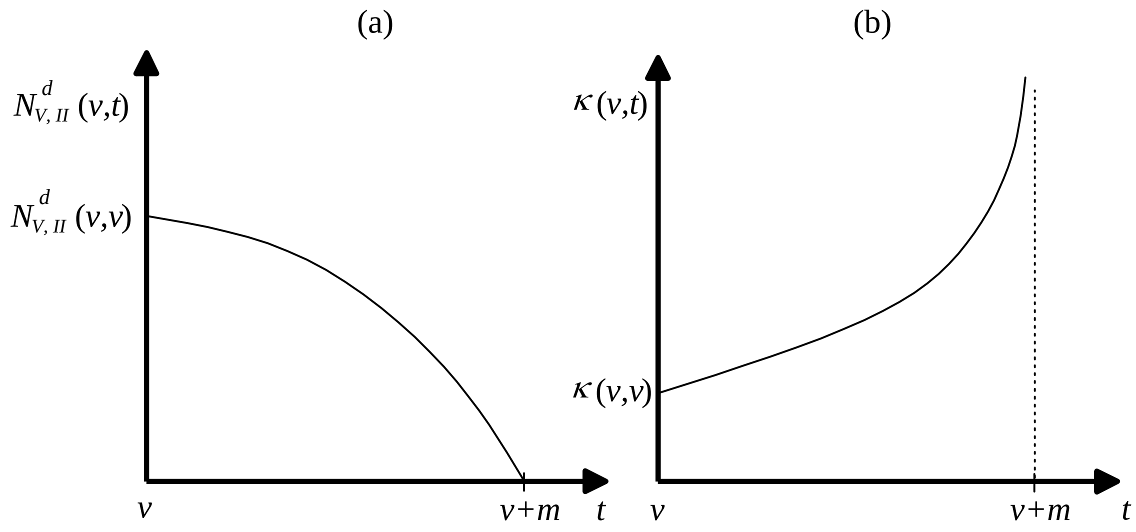

Figure 1: Demand for adaptable labor and human capital intensity of a certain vintage.

Figure 1 (a) shows that the demand for adaptable labor declines as a vintage ages. Due to condition (14) the wage rate of adaptable labor equals the marginal productivity of adaptable labor and must be equal across vintages. At the same time, technological progress increases the productivity of adaptable workers in younger vintages. Therefore, the number of adaptable workers employed decreases as vintages age. When a technology is discarded after m^* vintages, the demand for adaptable labor is positive.[28] The economic worklife of adaptable workers has not yet ended – they may continue to work until the statutory retirement age.

Unlike adaptable workers, labor using vintage specific skills is tied to a specific technology. Since the input of adaptable labor decreases as vintages age, $\kappa(v,t)$, the ratio of vintage specific human capital to adaptable workers, increases (see Figure 1 (b). Accordingly, the productivity of vintage specific human capital decreases as technologies become older. The rental rate of vintage specific human capital is denoted by $r_V(v,t)$. Since r_V is determined as residual income, lower output and higher wages for adaptable workers imply that r_V will decrease as a vintage ages.

[28] See Lemma 8.

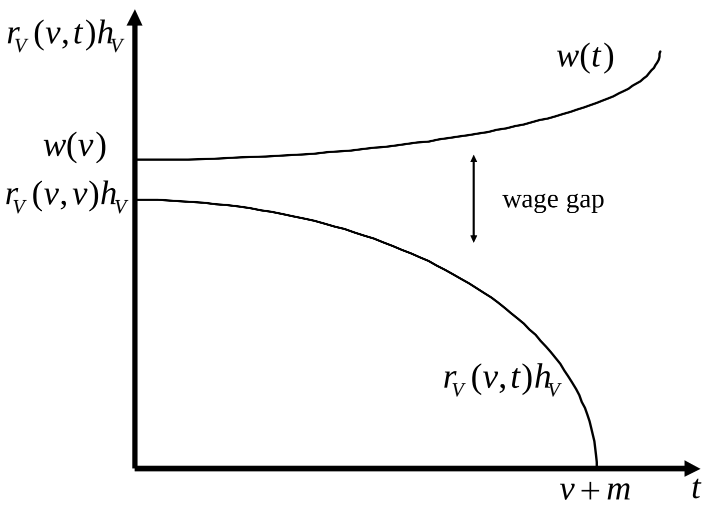

Figure 2: Income of vintage-specific human capital and adaptable labor

Thus, the income of workers using vintage specific skills decreases as they become older. The wage gap increases. These results are exactly in line with the findings by Violante (2002, p. 302). "... workers learn vintage specific skills and, when separating, they can only partially transfer their skills across machines. A technological acceleration has two effects. First, it reduces skill transferability thereby increasing wage losses upon separation. ...". Even more, a technology and human capital vintage is scrapped when the income of these workers is zero ($r_V(v, v+m) = 0$, see Figure 2). Full depreciation of human capital determines the economic retirement age of these workers.

Unemployment of Older Labor and Statutory Retirement Age: Unlike the statutory retirement age the effective economic retirement age m^* is endogenous and *not* under direct political control. A politically determined increase in the statutory retirement age T can only be a successful strategy as long as the statutory retirement age is below the economically determined time of full depreciation of vintage human capital ($T < m^*$). Full depreciation of vintage specific human capital means that even for a zero wage these workers are not employable. The reason for this market failure is a rigidity generated by the vintage structure of the technology, which lies in the past of the appearance of this market problem. Hence the described mechanism generates unemployment of older workers. The unemployment rate due to this kind of mechanism is[29]

$$u(t) = \frac{U(t)}{N^s} = \left(1 - \frac{m}{T}\right)(1 - q). \tag{18}$$

As a discarded vintage will never be employed again, the scrapping time also determines the duration of unemployment at the end of the working life of this group of workers. Hence this approach also sheds some light on the widely observed long-term unemployment among older workers.

Technological Change and Economic Retirement: Like in most growth models technological progress is the most important variable determining growth. However, in this model technological progress has both growth and employment effects. Due to technological advances new technologies are introduced permanently. Therefore, workers permanently have to adjust their skills. When a technology is scrapped after m^* vintages, it is replaced by a new technology. Therefore, the higher the rate of technological progress, i.e. the faster technologies are introduced and old technologies are replaced by new technologies, the faster technologies become outdated:[30]

$$\frac{\partial m^*}{\partial \hat{\lambda}} < 0. \tag{19}$$

A higher rate of technological progress results in a shorter economic scrapping time m^*.[31] Thus, the last $T - m^*$ vintages of workers using vintage specific skills become unemployed. If a technology is discarded, these workers will only be able to use

[29] $U(t) = N_I^s - N_I^d + N_{II}^s - N_{II}^d, \quad u(t) = \frac{U(t)}{N^s} = \frac{(t-m)(1-q)N_V^s - (t-T)(1-q)N_V^s}{N^s} = (T-m)(1-q)\frac{N_V^s}{N^s}$

$u(t) = \left(1 - \frac{m}{T}\right)(1-q)$

[30] See Corollary 7.

[31] Theoretically a lower rate of technological progress leads to a later scrapping time. But the restricting $m^* \leq T$ has to be considered. If theoretically the model generates a solution $m^* > T$

outdated technologies. The reason is the external depreciation of vintage specific human capital.[32] Workers entering the labor market are able to use the most modern technology available at that time. However, a fraction of the labor force, namely labor with vintage specific human capital, is not versatile enough to adjust their skills to newer technologies because the present efforts of lifelong learning were not sufficiently effective for this group. Not only do decreasing earning profiles of older workers occur, in the worst case older workers - belonging to the oldest $T - m^*$ technologies - even have to leave the active workforce. These workers are not employable and retire when the productive value of vintage specific human capital approaches zero. The unemployment rate increases accordingly:

$$\frac{\partial u}{\partial \hat{\lambda}} = -\frac{(1-q)}{T}\frac{\partial m^*}{\partial \hat{\lambda}} > 0. \tag{20}$$

As the duration of unemployment is also determined by $T - m^*$ we will also obtain an increase in the duration of unemployment.

International Integration, Outsourcing and Endogenous Retirement: In this model increasing global competition leads to a reduction in the relative price of imports p. Due to the small country assumption the price of imports is an exogenous variable. Improving terms of trade ($dp < 0$) affects the scrapping time:[33]

$$\frac{\partial m^*}{\partial p} < 0. \tag{21}$$

If the price of imports decreases, the demand for intermediary imports, i.e. the level of outsourcing of domestic production in favor of international imported intermediary goods, will increase. Since intermediary imports can be substituted by domestic labor services, the demand for domestic resources decreases in all vintages.

the model has to be solved backward starting with $T = m^*$. Using the Kuhn-Tucker conditions it can be shown that in this case the profit maximization condition no longer holds.

[32] According to Neuman and Weiss (1995) we can distinguish between internal and external human capital depreciation. The internal depreciation of human capital – the loss of physical abilities and mental capacities – is not considered in the model.

[33] See Corollary 7.

Thus, the demand for adaptable labor combined with new technologies decreases in each vintage. Because a technology vintage is discarded, when adaptable labor is demanded by the newest m^* technologies, more vintages can now be combined with adaptable labor such that the scrapping time increases. This mechanism suggests that increasing international integration – with decreasing import prices ($dp < 0$) for most of the advanced economies – may also have positive effects for older workers in OECD countries. According to the model international specialization and outsourcing can help to slow down the speed of endogenous human capital depreciation and may reduce unemployment:

$$\frac{\partial u}{\partial p} = -\frac{(1-q)}{T}\frac{\partial m^*}{\partial p} > 0. \tag{22}$$

4 Education Policy

Up to this point we have discussed how different factors determine the endogenous retirement age from the demand side perspective of the labor market. A complex interaction between input factors and vintage technology determines the demand structure for different qualifications and hence the scrapping time of vintage specific human capital.

However, in addition to the structure of labor demand the structure of labor supply is also relevant. For a given vintage of people a two-stage education process generates a certain structure of skills and competencies during their working life. For this supply structure of labor not all labor may be employable until the statutory retirement age (the conditions are discussed above). Although the problem is essentially linked to the production technology used, an important question to answer is if and how education policy can solve the problem of a (too) rapid endogenous depreciation of vintage specific human capital. Can it help to align the economic and statutory retirement age of vintage specific human capital? In other words, how can we adjust the time of full depreciation of human capital vintages to match the politically desired age? Are there policy instruments that may affect the economic retirement age within the introduced two-stage system of education? Can education policy affect *unemployment* of older workers and *long-term unemployment* of this group?

Increasing Efforts in Initial Schooling: The described problem is caused by the depreciation process of vintage specific human capital. Skills learned during *initial schooling* are fully depreciated. Therefore we were able to follow the idea of improving *initial schooling* and hence the productivity of vintage specific skills to prolong the duration of usability of these skills. In the model this policy would be an increase in the fraction of GDP τ_1 spend on *initial schooling*.

However, looking at the model's results we can see that raising the fraction of GDP spent on *initial schooling* does not affect the scrapping time of this sort of human capital and hence does not help to align the retirement age with the statutory level:[34]

$$\frac{\partial m^*}{\partial \tau_1} = 0. \tag{23}$$

An improvement in *initial schooling* fails to reduce the rate of withdrawal from the labor market of older workers. While resources spent on *initial schooling* affect the level of a number of different variables (especially the wage of workers with vintage specific skills), they do not affect the structure of growth rates that determine scrapping time.

Raising τ_1 leads to an increasing supply of vintage specific human capital such that human capital intensity increases as the price of vintage specific human capital, the quasi-rent of each unit of human capital, decreases. There are two effects. First, even though scrapping time is not affected, workers with vintage specific skills achieve higher earnings for the time they are employed. Due to better education the rental income curve of a worker's vintage specific human capital $r_V(v,t)h_V$ (see Figure 2) moves upward. Second, vintage output and thus the aggregate output increases. Higher earnings and higher aggregate output may therefore help to im-

[34]See Corollary 7.

prove the financial health of the pension system. However, this is a secondary effect. Higher spending on *initial schooling* is neither a feasible policy option for decreasing the rate or duration of unemployment of older workers nor is it able to adjust the economic retirement age:

$$\frac{\partial u}{\partial \tau_1} = 0. \tag{24}$$

Increasing Efforts in *Lifelong Learning Programs:* Since increasing spending on standard initial schooling cannot solve the problem of synchronizing the statutory and the economic retirement age, is there any effect from higher spending in *lifelong learning programs*? Looking again at the model's results we can see that additional spending in *lifelong learning programs* does not affect the scrapping time either:[35]

$$\frac{\partial m^*}{\partial \tau_2} = 0. \tag{25}$$

Once again, the structure of the growth rates in the steady state is not affected.[36]

However, higher spending on *lifelong learning programs* may lead to a higher success rate of such programs and hence will help more people acquire the ability to adjust to new technologies. Fewer people are expected to be left with vintage specific skills only. Even if $m^* < T$ remains we obtain a decreasing unemployment rate

$$\frac{\partial u}{\partial \tau_2} = -\left(1 - \frac{m}{T}\right)\frac{\partial q}{\partial \tau_2} < 0. \tag{26}$$

Therefore, the problem is not solved but it becomes less severe on aggregate. Investments in *lifelong learning programs* providing flexible abilities should therefore complement the introduction of a higher statutory retirement age.

[35] See Corollary 7.

[36] However, if the different labor services in (7) are complements, there may also be an effect on scrapping time because an increasing productivity of skilled labor would only partially be offset by an increase in wages. Thus, the employment of additional workers with inflexible skills would be possible and, as a result, the scrapping time of technology vintages would increase.

However, proposing additional marginal investments for *lifelong learning programs* addresses not only technological conditions. The fraction of labor targeted by extended programs is the group where existing *lifelong learning programs* did not work so far. Therefore, it can be expected that learning capabilities of this group are at the lower end of society's capability distribution and hence limited. Therefore, the cost of upgrading abilities can be expected to increase significantly. For this group the problem of high training in return for small ability improvement becomes even more obvious when we think of these people as older workers. As a result, additional investment in *lifelong learning programs* may not pay off. Only as long as marginal spending on lifelong learning programs leads to a sufficient number of additional adaptable workers, such that the generated additional income exceeds the spending, will this education policy be efficient. Therefore, if the learning capabilities of the remaining workers (with vintage specific human capital only) is too low, unemployment may be an efficient solution.

5 Conclusion

The employability of an aging population in a world of continuous technical change has become one of the major concerns in today's policy debate. Among others, the European Union in its Lisbon Strategy emphasizes that maintaining employment opportunities even for older workers is crucial for the EU's competitiveness and social cohesion. At the same time, many governments plan to increase the statutory retirement age in order to stabilize their pension systems. However, in many countries high net withdrawal rates among older workers can be observed.

Our paper intends to explain why the economic – or effective– retirement age may fall short of statutory retirement, thereby generating unemployment among older worker cohorts. We argue that the economic retirement age has to be considered endogenous due to the depreciation of vintage specific human capital. Furthermore, we identify demand side factors explaining a decreasing wage profile and early retirement, and investigate the effectiveness of policy measures on the supply side. In particular we analyze whether improvements in the schooling system can synchronise statutory and effective retirement or stabilize employment levels of an aging generation.

To analyze the economic determination of retirement we use a putty-putty vintage model with heterogeneous agents. Schooling is organized in a sequential two stage education system, namely an *initial schooling program* and a *lifelong learning program*. Initial schooling equips workers with skills that enable them to use the most recent technology vintage available at the time when a new generation of workers enter the market. Hence initial schooling is a technology vintage specific training. The subsequent lifelong learning program enables workers to switch to new technologies at any time. Lifelong learning is a resource-absorbing schooling program that helps to adjust aging labor to a changing technical environment.

The two stage education system together with differences in learning capabilities of the workers generates an endogenous structure of the labor force. Even all workers participate in lifelong learning the resources spent on this program determine the fraction of the labor force that can successfully upgrade skills and hence adapt to new technologies. The fraction of labor that is not successful at the present volume of spending in this program is bound to the technology vintage of their initial schooling period. As shown in the model, upgrading skills and handling new technologies has a strong effect on the income and employability of a population vintage when the vintage ages.

While education is often regarded as the cure for early human capital depreciation, our analysis suggests that the effects of education and even of a lifelong learning program are limited with respect to the duration of employability. More specifically, better initial schooling will affect income levels and the distribution of income but will have no effect on the time path of human capital depreciation and the economic retirement age. Even a lifelong learning program does not solve the problem of synchronizing the economic and the statutory retirement age. From our model we learn that lifelong learning helps to increase the individual probability of finding a job when people grow older and hence reduces the unemployment rate of an aging generation. However, lifelong learning will not affect the speed of the depreciation process. When deciding to either increase statutory retirement age or improve lifelong learning institutions, the latter should be the preferred first step as long as the investments in the program still have a sufficiently positive return. These investments will lead to positive effects on unemployment rates and generate contributions to the pension system. If this policy alone is not sufficient the statutory retirement age may be increased. Even if the lifelong learning program cannot synchronize the effective and statutory retirement for workers with lower learning capabilities (vintage skills), a smaller fraction of labor will not reach the statutory retirement age and drop out of the market before.

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Appendix

Lemma 1 *In steady–state it is* $\hat{\alpha}_v = \hat{\beta}_v = \hat{\delta}_v = \hat{\gamma}_v = 0.$

Proof: Equation (5) is equivalent to

$$\begin{aligned} (1-\gamma)\left(\frac{Y_V(t,t)}{S(t,t)}\right)^{\rho_1} &= \delta_t \left(\frac{Y_V(t,t)}{S(t,t)}\right)^{\rho_2} \\ \Longleftrightarrow \qquad \delta_t = (1-\gamma)\left(\frac{Y_V(t,t)}{S(t,t)}\right)^{\rho_1-\rho_2}. \end{aligned}$$

It follows directly $\hat{\delta}_t = 0$. Using condition (6) leads to

$$\begin{aligned} Y_V(t,t)^{-\rho_2} &= \gamma_t Im(t,t)^{-\rho_2} + \delta_t S(t,t)^{-\rho_2} \\ \Longleftrightarrow \qquad \gamma_t &= \frac{Y_V(t,t)^{-\rho_2} - \delta_t S(t,t)^{-\rho_2}}{Im(t,t)^{-\rho_2}} \end{aligned}$$

and it follows $\hat{\gamma}_t = 0$. Equation (5) can be transformed to

$$\begin{aligned} \alpha \frac{S(t,t)}{N^d_{V,II}(t,t)} &= \alpha_t \lambda_V(t)^{-\rho_4} \left(\frac{S(t,t)}{N^d_{V,II}(t,t)}\right)^{\rho_4} \\ \Longleftrightarrow \qquad \alpha_t &= \alpha \lambda_V(t)^{\rho_4} \left(\frac{S(t,t)}{N^d_{V,II}(t,t)}\right)^{-\rho_4}. \end{aligned}$$

The last equation implies $\hat{\alpha}_t = 0$. Using condition (10) leads to

$$\begin{aligned} S(t,t)^{-\rho_4} &= \alpha_t \Lambda_V(t,t)^{-\rho_4} + \beta_t H_V(t,t)^{-\rho_4} \\ \Longleftrightarrow \qquad \beta_t &= \frac{S(t,t)^{-\rho_4} - \alpha_t \Lambda_V(t,t)^{-\rho_4}}{H_V(t,t)^{-\rho_4}} \end{aligned}$$

and it follows $\hat{\beta}_t = 0$. □

Lemma 2 *If for the time of installing a new technology condition (12) holds, it will be* $im(t,t) = y_V(t,t)\left(\frac{\gamma}{p}\right)^{\frac{1}{1+\rho_1}}$ *and*

$$y_V(t,t) = c_1 s(t,t) \qquad \textit{with} \qquad c_1 := (1-\gamma)^{-\frac{1}{\rho_1}}\left(1-\gamma^{\frac{1}{1+\rho_1}} p^{\frac{\rho_1}{1+\rho_1}}\right)^{\frac{1}{\rho_1}}.$$

As for an installed technology equation (13) hold, it follows $im(v,t) = y_V(v,t)\left(\frac{\gamma_v}{p}\right)^{\frac{1}{1+\rho_2}}$ *and*

$$y_V(v,t) = c_2 s(v,t) \qquad \textit{with} \qquad c_2 := (\delta_v)^{-\frac{1}{\rho_2}}\left(1-\gamma_v^{\frac{1}{1+\rho_2}} p^{\frac{\rho_2}{1+\rho_2}}\right)^{\frac{1}{\rho_2}}.$$

Proof: It is

$$\frac{\partial y_V(t,t)}{\partial im(t,t)} = p \iff y_V(t,t) = im(t,t)\left(\frac{p}{\gamma}\right)^{\frac{1}{1+\rho_1}} \iff im(t,t) = y_V(t,t)\left(\frac{\gamma}{p}\right)^{\frac{1}{1+\rho_1}}.$$

It follows

$$\begin{aligned}
& \left(\frac{y_V(t,t)}{s(t,t)}\right)^{-\rho_1} = \gamma\left(\frac{im(t,t)}{s(t,t)}\right)^{-\rho_1} + (1-\gamma) = \gamma^{\frac{1}{1+\rho_1}} p^{\frac{\rho_1}{1+\rho_1}} \left(\frac{y_V(t,t)}{s(t,t)}\right)^{-\rho_1} + (1-\gamma) \\
\iff \quad & \left(\frac{y_V(t,t)}{s(t,t)}\right)^{-\rho_1} = \frac{1-\gamma}{1-\gamma^{\frac{1}{1+\rho_1}} p^{\frac{\rho_1}{1+\rho_1}}} \\
\iff \quad & y_V(t,t) = s(t,t)()^{-\frac{1}{\rho_1}} \left(1-\gamma^{\frac{1}{1+\rho_1}} p^{\frac{\rho_1}{1+\rho_1}}\right)^{\frac{1}{\rho_1}}.
\end{aligned}$$

There is a linear relation between $im(t,t)$ and $s(t,t)$:

$$\begin{aligned}
& im(t,t)\left(\frac{p}{\gamma}\right)^{\frac{1}{1+\rho_1}} = s(t,t)(1-\gamma)^{-\frac{1}{\rho_1}} \left(1-\gamma^{\frac{1}{1+\rho_1}} p^{\frac{\rho_1}{1+\rho_1}}\right)^{\frac{1}{\rho_1}} \\
\iff \quad & im(t,t) = s(t,t)(1-\gamma)^{-\frac{1}{\rho_1}} \left(1-\gamma^{\frac{1}{1+\rho_1}} p^{\frac{\rho_1}{1+\rho_1}}\right)\left(\frac{\gamma}{p}\right)^{\frac{1}{1+\rho_1}}.
\end{aligned}$$

The first statement of the lemma will follow, if this result is inserted in the production function:

$$\begin{aligned}
y_V(t,t) &= \left(\gamma\left[\left(\frac{1-\gamma}{1-\gamma^{\frac{1}{1+\rho_1}} p^{\frac{\rho_1}{1+\rho_1}}}\right)^{-\frac{1}{\rho_1}} \left(\frac{\gamma}{p}\right)^{\frac{1}{1+\rho_1}}\right]^{-\rho_1} + (1-\gamma)\right)^{-\frac{1}{\rho_1}} s(t,t) \\
&= \left(\left(\frac{1-\gamma}{1-\gamma^{\frac{1}{1+\rho_1}} p^{\frac{\rho_1}{1+\rho_1}}}\right) \gamma^{\frac{1}{1+\rho_1}} p^{\frac{\rho_1}{1+\rho_1}} + (1-\gamma)\right)^{-\frac{1}{\rho_1}} s(t,t) \\
&= \underbrace{\left(\frac{1-\gamma}{1-\gamma^{\frac{1}{1+\rho_1}} p^{\frac{\rho_1}{1+\rho_1}}}\right)^{-\frac{1}{\rho_1}}}_{=:c_1} s(t,t).
\end{aligned}$$

The second part of the proof is organized in the same way. It is

$$\frac{\partial y_V(v,t)}{\partial im(v,t)} = p \iff y_V(v,t) = im(v,t)\left(\frac{p}{\gamma_v}\right)^{\frac{1}{1+\rho_2}} \iff im(v,t) = y_V(v,t)\left(\frac{\gamma_v}{p}\right)^{\frac{1}{1+\rho_2}}.$$

It follows

$$\left(\frac{y_V(v,t)}{s(v,t)}\right)^{-\rho_2} = \gamma_v \left(\frac{im(v,t)}{s(v,t)}\right)^{-\rho_2} + \delta_v = \gamma_v^{\frac{1}{1+\rho_2}} p^{\frac{\rho_2}{1+\rho_2}} \left(\frac{y_V(v,t)}{s(v,t)}\right)^{-\rho_2} + \delta_v$$

$$\iff \quad \left(\frac{y_V(v,t)}{s(v,t)}\right)^{-\rho_2} = \frac{\delta_v}{1-\gamma_v^{\frac{1}{1+\rho_2}} p^{\frac{\rho_2}{1+\rho_2}}}$$

$$\iff \quad y_V(v,t) = s(v,t)\left(\frac{\delta_v}{1-\gamma_v^{\frac{1}{1+\rho_2}} p^{\frac{\rho_2}{1+\rho_2}}}\right)^{-\frac{1}{\rho_2}}.$$

As above we get for $im(v,t)$ and $s(v,t)$:

$$im(v,t)\left(\frac{p}{\gamma_v}\right)^{\frac{1}{1+\rho_2}} = s(v,t)\left(\frac{\delta_v}{1-\gamma_v^{\frac{1}{1+\rho_2}} p^{\frac{\rho_2}{1+\rho_2}}}\right)^{-\frac{1}{\rho_2}}$$

$$\iff \quad im(v,t) = s(v,t)\left(\frac{\delta_v}{1-\gamma_v^{\frac{1}{1+\rho_2}} p^{\frac{\rho_2}{1+\rho_2}}}\right)^{-\frac{1}{\rho_2}} \left(\frac{\gamma_v}{p}\right)^{\frac{1}{1+\rho_2}}.$$

Inserting this result into the production function (4), it follows the second statement:

$$\begin{aligned}
y_V(v,t) &= \left(\gamma_v \left[\left(\frac{\delta_v}{1-\gamma_v^{\frac{1}{1+\rho_2}} p^{\frac{\rho_2}{1+\rho_2}}}\right)^{-\frac{1}{\rho_2}} \left(\frac{\gamma_v}{p}\right)^{\frac{1}{1+\rho_2}}\right]^{-\rho_2} + \delta_v\right)^{-\frac{1}{\rho_2}} s(v,t) \\
&= \left(\left(\frac{\delta_v}{1-\gamma_v^{\frac{1}{1+\rho_2}} p^{\frac{\rho_2}{1+\rho_2}}}\right) \gamma_v^{\frac{1}{1+\rho_2}} p^{\frac{\rho_2}{1+\rho_2}} + \delta_v\right)^{-\frac{1}{\rho_2}} s(v,t) \\
&= \underbrace{\left(\frac{\delta_v}{1-\gamma_v^{\frac{1}{1+\rho_2}} p^{\frac{\rho_2}{1+\rho_2}}}\right)^{-\frac{1}{\rho_2}}}_{=:c_2} s(v,t).
\end{aligned}$$

Observe that c_2 is constant, because γ_v and δ_v are constant due to Lemma 1. □

Corollary 3 *For the wage rate of flexible labor the following condition holds:* $w(t) = \alpha y_V(t,t)$

Proof: Using Lemma 2 we get the wage when writing equation (14) for $v = t$:

$$\begin{aligned}
w(t) &= \frac{\partial Y_V(t,t)}{\partial N^d_{V,II}(t,t)} = \frac{\partial Y_V(t,t)}{\partial S(t,t)} \frac{\partial S(t,t)}{\partial N^d_{V,II}(t,t)} \\
&= c_1 \frac{\partial S(t,t)}{\partial N^d_{V,II}(t,t)} = c_1 \alpha \frac{S(t,t)}{N^d_{V,II}(t,t)} = \alpha y_V(t,t).
\end{aligned}$$

□

Corollary 4 *For the output in intensity form we get*

$$y_V(v,t) = c_2 \left(\frac{w(t)}{c_2 \alpha_v \lambda_V(v)^{-\rho_4}} \right)^{\frac{1}{1+\rho_4}}$$

and therefore

$$y_V(v,t) = y_V(0,0) e^{\frac{1}{1+\rho_4}\hat{\lambda} t} e^{\frac{\rho_4}{1+\rho_4}\hat{\lambda} v}.$$

For the imports it is

$$im(v,t) = im(0,0) e^{\frac{1}{1+\rho_4}\hat{\lambda} t} e^{\frac{\rho_4}{1+\rho_4}\hat{\lambda} v}.$$

Proof: Using Lemma 2 equation (14) for $v < t$ leads to:

$$\begin{aligned} w(t) &= \frac{\partial Y_V(v,t)}{\partial N^d_{V,II}(v,t)} = \frac{\partial Y_V(v,t)}{\partial S(v,t)} \frac{\partial S(v,t)}{\partial N^d_{V,II}(v,t)} \\ &= c_2 \frac{\partial S(v,t)}{\partial N^d_{V,II}(v,t)} = c_2 \alpha_V \lambda_V(v)^{-\rho_4} \left(\alpha_v \lambda_V(v)^{-\rho_4} + \beta_v \kappa(v,t)^{-\rho_4} \right)^{-\frac{1+\rho_4}{\rho_4}} \\ \iff \quad \kappa(v,t)^{-\rho_4} &= \frac{\left(\frac{w(t)}{c_2 \alpha_v \lambda_V(v)^{-\rho_4}} \right)^{-\frac{\rho_4}{1+\rho_4}} - \alpha_v \lambda_V(v)^{-\rho_4}}{\beta_v}. \end{aligned}$$

Inserting this result into the production function (8) and using Lemma 2 we get

$$y_V(v,t) = c_2 \left(\frac{w(t)}{c_2 \alpha_v \lambda_V(v)^{-\rho_4}} \right)^{\frac{1}{1+\rho_4}}.$$

The growth rate of $y_V(v,t)$ follow from $\hat{w} = \hat{\lambda}_V = \hat{\lambda}$ and $\hat{\alpha} = 0$. The statement for the imports follows from Lemma 2, since $im(v,t) = c_3 y_V(v,t)$ with $c_3 = \left(\frac{\gamma_v}{p}\right)^{\frac{1}{1+\rho_2}}$ holds. □

Corollary 5 *Equation (13) for $v = t$ is equivalent to equation (12), i.e. it is* $\frac{\partial y_V(t,t)}{\partial im(t,t)} = p$.

Proof: The statement follows directly, if $y_V(t,v) = y_V(t,t) e^{-\frac{1}{1+\rho_4}}$ and $im(t,v) = im(t,t) e^{-\frac{1}{1+\rho_4}}$ are inserted in equation (12). It has to be considered that only the second part of Lemma 2 is used. □

Theorem 6 *The scrapping time in steady–state is*

$$m^* = \frac{\ln\left[\left(1-\left(\frac{\gamma}{p}\right)^{\frac{1}{1+\rho_1}}\right)^{-1}\alpha\right]}{-\hat{\lambda}}\frac{1+\rho_4}{\rho_4}.$$

Proof: A vintage is discarded, if returns of the vintage–specific human capital are zero:

$$y_V(t-m^*,t) - im(t-m^*,t) = w(t).$$

In Lemma 4 it is proved that $y_V(t-m,t) = y_V(t,t)e^{-\frac{\rho_4}{1+\rho_4}\hat{\lambda}m}$, following Lemma 2 and Corollary 4 we get $im(t-m,t) = im(t,t)e^{-\frac{\rho_4}{1+\rho_4}\hat{\lambda}m} = y_V(t,t)\left(\frac{\gamma}{p}\right)^{\frac{1}{1+\rho_1}} e^{-\frac{\rho_4}{1+\rho_4}\hat{\lambda}m}$ and following Lemma 3 it is $w(t) = \alpha y_V(t,t)$. Eliminating $y_V(t,t)$, we get:

$$e^{-\frac{\rho_4}{1+\rho_4}\hat{\lambda}m^*}\left(1-\left(\frac{\gamma}{p}\right)^{\frac{1}{1+\rho_1}}\right) = \alpha$$

$$\iff \qquad m^* = \frac{\ln\left[\left(1-\left(\frac{\gamma}{p}\right)^{\frac{1}{1+\rho_1}}\right)^{-1}\alpha\right]}{-\hat{\lambda}}\frac{1+\rho_4}{\rho_4}.$$

□

Corollary 7 *In steady–state it is:*

$$\begin{array}{llll}
\frac{\partial m^*}{\partial \hat{\lambda}} < 0, & \frac{\partial m^*}{\partial \tau_1} = 0, & \frac{\partial m^*}{\partial \alpha} > 0, & \frac{\partial m^*}{\partial p} < 0, \\
\frac{\partial m^*}{\partial \gamma} > 0, & \frac{\partial m^*}{\partial \rho_1} < 0, & \frac{\partial m^*}{\partial \rho_2} = 0, & \frac{\partial m^*}{\partial \rho_4} < 0,
\end{array}$$

Lemma 8 *It is $N^d_{V,II}(t,t+m^*) > 0$ and $Y_V(t,t+m^*) > 0$.*

Proof: It is $\alpha_v\lambda_V(t)^{-\rho_4} = \alpha\left(\frac{y_V(t,t)}{c_2}\right)^{-\rho_4}$ and $y_V(v,v) = y_V(t,t)e^{\hat{\lambda}(v-t)}$. Using these conditions we get

$$\begin{aligned}
\frac{N^d_{V,II}(t,v)}{N^d_{V,II}(t,t)} &= \left(\frac{\left(\frac{w(v)}{c_2\alpha_t\lambda_V(t)^{-\rho_4}}\right)^{-\frac{\rho_4}{1+\rho_4}} - \alpha_t\lambda_V(t)^{-\rho_4}}{\left(\frac{w(t)}{c_2\alpha_t\lambda_V(t)^{-\rho_4}}\right)^{-\frac{\rho_4}{1+\rho_4}} - \alpha_t\lambda_V(t)^{-\rho_4}}\right)^{\frac{1}{\rho_4}} \\
&= \left(\frac{c_2^{\rho_4}y_V(t,t)^{-\rho_4}e^{-\hat{\lambda}(v-t)\frac{\rho_4}{1+\rho_4}} - \alpha y_V(t,t)^{-\rho_4}c_2^{\rho_4}}{c_2^{\rho_4}y_V(t,t)^{-\rho_4} - \alpha y_V(t,t)^{-\rho_4}c_2^{\rho_4}}\right)^{\frac{1}{\rho_4}} \\
&= \left(\frac{e^{-\hat{\lambda}(v-t)\frac{\rho_4}{1+\rho_4}} - \alpha}{1-\alpha}\right)^{\frac{1}{\rho_4}}.
\end{aligned}$$

It follows that

$$\begin{aligned} N^d_{V,II}(t,v) = 0 \quad & \Longleftrightarrow \quad v - t = \frac{\ln\alpha}{-\hat{\lambda}}\frac{1+\rho_4}{\rho_4} > m^* \\ & \Longleftrightarrow \quad \ln\alpha < \ln\left[\left(1 - \left(\frac{\gamma}{p}\right)^{\frac{1}{1+\rho_1}}\right)^{-1}\alpha\right] \\ & \Longleftrightarrow \quad 1 < \left(1 - \left(\frac{\gamma}{p}\right)^{\frac{1}{1+\rho_1}}\right)^{-1}. \end{aligned}$$

Since this condition holds, the statement follows. □

Lemma 9 *The aggregate output Y or $y = \frac{Y}{N^d_{V,II}}$ grows with the rate of technological progress in steady–state.*

Proof: Using Lemma 8 it is

$$\begin{aligned} y &= \int_{t-m}^{t} \frac{N^d_{V,II}(v,t)}{N^d_{V,II}(t,t)} y_V(v,t)dv \\ &= \int_{t-m}^{t} \underbrace{\left(\frac{1-(1-\alpha)e^{\hat{\lambda}(t-v)\frac{\rho}{1+\rho}}}{\alpha}\right)^{\frac{1}{\rho}}}_{=:h(v,t)} y_V(v,v)dv. \end{aligned}$$

Using the partial integration method and $\frac{\partial h}{\partial v}(v,t) = -\frac{\partial h}{\partial t}(v,t)$ we get

$$y = \frac{1}{\hat{\lambda}}\left[y_V(t,t)\left(h(t,t) - h(t-m,t)e^{-\hat{\lambda}m}\right) + \int_{t-m}^{t}\frac{\partial h}{\partial t}(v,t)y_V(v,v)dv\right].$$

We differentiate this equation

$$\dot{y} = y_V(t,t)\left(h(t,t) - h(t-m,t)e^{-\hat{\lambda}m}\right) + \int_{t-m}^{t}\frac{\partial h}{\partial t}(v,t)y_V(v,v)dv$$

and together it is $\frac{\dot{y}}{y} = \hat{\lambda}$. □

Theorem 10 *The equations (1) and (11) determine uniquely the human capital intensity $\kappa(0,0)$ and the quasi–rent r, which is constant in steady–state, if the following condition holds*

$$\tau_1 < \tau_1^{max} := \frac{(1-\alpha)\int_0^m \frac{N^d_{V,II}(0,v)}{N^d_{V,II}(0,0)} e^{\frac{1}{1+\rho_4}\hat{\lambda}v}dv}{(1-q(\tau_2 Y))\int_{-m}^0 \frac{N^d_{V,II}(v,0)}{N^d_{V,II}(0,0)} e^{\frac{\rho_4}{1+\rho_4}\hat{\lambda}v}dv}.$$

Proof: In steady–state equation (1) can be written as

$$\kappa(0,0) = \tau_1 y(0) = \tau_1(1-q(\tau_2 Y)) \int_{-m}^{m} \frac{N^d_{V,II}(v,0)}{N^d_{V,II}(0,0)} y_V(v,0) dv$$

$$\iff \quad f_1(r) := \frac{\kappa(0,0)}{y_V(0,0)} = \tau_1(1-q(\tau_2 Y)) \int_{-m}^{0} \frac{N^d_{V,II}(v,0)}{N^d_{V,II}(0,0)} e^{\frac{\rho_4}{1+\rho_4}\hat{\lambda} v} dv.$$

It is $\frac{\partial y_V(0,0)}{\partial \kappa(0,0)} = (1-\alpha)\frac{y_V(0,0)}{\kappa(0,0)}$ and due to Corollary 4 $y_V(0,v) = y_V(0,0)e^{\frac{1}{1+\rho_4}\hat{\lambda} v}$. Therefore, equation (11) can be written as:

$$f_2(r) := \frac{\kappa(0,0)}{y_V(0,0)} = (1-\alpha) \int_{0}^{m} \frac{N^d_{V,II}(0,v)}{N^d_{V,II}(0,0)} e^{\frac{1}{1+\rho_4}\hat{\lambda} v} e^{-rv} dv.$$

$f_1(r)$ is a constant function and $f_2(r)$ a monotone increasing function. The uniqueness follows. The existence is fulfilled, if

$$f_1(0) < f_2(0) \quad \iff \quad \tau_1 < \frac{(1-\alpha)\int_0^m \frac{N^d_{V,II}(0,v)}{N^d_{V,II}(0,0)} e^{\frac{1}{1+\rho_4}\hat{\lambda} v} dv}{(1-q(\tau_2 Y))\int_{-m}^0 \frac{N^d_{V,II}(v,0)}{N^d_{V,II}(0,0)} e^{\frac{\rho_4}{1+\rho_4}\hat{\lambda} v} dv} = \tau_1^{max}$$

holds. □

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